friday:

mslc: webwork workshop @ 12:30, 1:30, 2:30, 3:30, 4:30 in SEL 040
webwork homework vi due @ 11:55 pm

monday:

webwork extra credit ii due @ 6:00 am

tuesday, 24 november:

§ 9.3 - separable equations
§ 9.4 - exponential growth and decay

thursday, 26 november:

no school (thanksgiving)

tuesday, 1 december:

review for final
quiz v: §§ 9.1, 9.3

thursday, 3 december:

review for final
homework viii due: 9.1.4, 9.1.12, 9.3.12, 9.3.36, 9.4.4, 9.4.14.

friday, 4 december:

mslc: webwork workshop @ 12:30, 1:30, 2:30, 3:30, 4:30 in SEL 040
webwork homework vii due @ 11:55 pm

math helps us understand the world

example

Jill went up the hill and picked a dozen apples to make two apple pies for thanksgiving. Jack saw the apples in the kitchen and started eating them. By the time Jill caught him, there were only eight apples left. How many apples did Jack eat?

http://upload.wikimedia.org/wikipedia/commons/b/b5/Malus_Goldrenette_F._v._Berlepsch.jpg

example

Let $x =$ the number of apples Jack ate

Then we have the equation

$$12 - x = 8,$$

which we can solve to find $x = 4$.

That is, Jack ate four apples.

An equation is simply a statement that two things are equal.

In this case, the equation tells us something about x. (We can solve for x.)

If we did it correctly, we can plug the x we found into the original equation and get a true statement.

http://upload.wikimedia.org/wikipedia/commons/b/b5/Malus_Goldrenette_F._v._Berlepsch.jpg

differential equations

A differential equation is an equation that contains an unknown function and one or more of its derivatives.

An ordinary differential equation (or ODE) is a differential equation where the unknown function is a function of only one independent variable. (For example, the function may be a function of time or position but not both.)

We will only talk about ODEs because you only know one variable calculus.

examples

The following are differential equations:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -y$$

First equation has solutions:

y=C e^x

For second:

y=C sin(x) + D cos(x)

Can you think of any solutions?

example

Show

$$x = t\,e^t \qquad \text{and} \qquad x = e^t$$

are both solutions to

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} - 2\,\frac{\mathrm{d}x}{\mathrm{d}t} + x = 0$$

Take the appropriate derivatives and plug in.

(This is the exact same thing we'd do to check our answers in algebra.)

but why do we care?

We care because often things change over time. How they change is a statement about derivatives.

Hooke's Law

The force f required to maintain a spring stretched x units beyond its natural length is proportional to x:

$$f = k\,x$$

for some constant k.

We first encountered Hooke's law in lecture 8 on work.

If we need this much force to maintain position, then the spring is exerting a force of -kx.

springs

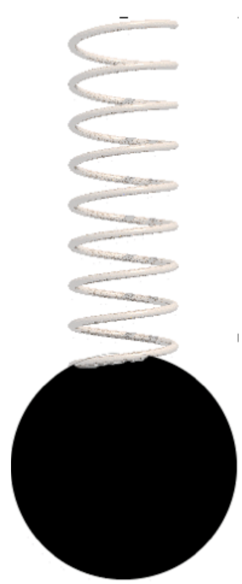
A spring stretched by an amount x exerts a force of $f = -\,k\,x$.

But by Newton's second law, $f = m\,a$, where $a = \mathrm{d}^2 x/\mathrm{d}t^2$.

Thus the motion of a mass on a spring can be described by

$$m\,\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -k\,x$$

Here we're ignoring any effect due to gravity. If we were including gravity, there would be an mg term on the right.



Newton's law of cooling

"The rate of cooling of an object is proportional to the temperature difference between the object and its surroundings."

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k \left(T_{\mathrm{outside}} - T\right)$$

Quote is from Stewart, section 9.3.

This is one tool for estimating how long something has been allowed to cool.

population growth

How could we predict the number of bacteria grown in culture after a certain amount of time had elapsed?

One reasonable model would be to assume the population grows at a rate proportional to the current population.

$$\frac{\mathrm{d}P}{\mathrm{d}t} = k\,P$$

This is called the law of natural growth. It applies when the population is not limited by resources or predation. The same differential equation applies for radioactive decay when $k<0$.

population growth

We can solve the differential equation for the law of natural growth:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = k\,P$$

by the method of separation of variables: get all the P's on one side, all the t's on the other, then integrate:

$$\frac{\mathrm{d}P}{P} = k\,\mathrm{d}t$$

Solving for P, we find

P = e^(kt)

coming soon

- read §§ 9.3, 9.4 for tuesday
- webwork vi due friday
- extra credit project 2 due monday
- extra credit project 3 due on 7 december